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During the 1991-1992 academic year the Institute for Advanced Study conducted a program in applied mathematics with special emphasis on computational fluid dynamics. This was the first time since the days of von Neumann that the Institute has had a major presence in applied mathematics. The senior methematicians who formed the core of this program were A Chorin, B Engquist, A Majda, G Papanicolaou and V Rokhlin. There were a total of about fifteen members participating in the program. AFOSR helped make this ambitious program possible by providing partial support for George Papanicolaou and Michael Shelly. They were both major participants in the program and interacted intensively with their colleagues.

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Final Report

During the 1991-92 academic year the Institute for Advanced Study conducted a program in applied mathematics with special emphasis on computational fluid dynamics. This was the first time since the days of von Neumann that the Institute has had a major presence in applied mathematics. The senior mathematicians who formed the core of this program were A. Chorin, B. Engquist, A. Majda, G. Papanicolaou and V. Rokhlin. There were a total of about fifteen members participating in the program.

The Air Force Office of Scientific Research helped make this ambitious program possible by providing partial support for George Papanicolaou and Michael Shelley. They were both major participants in the program and interacted intensively with their colleagues. Enclosed is their description of some of the work they did at the Institute.

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Research Report

George Papanicolaou

My primary research activity this year was the study of convection enhanced diffusion. I worked closely with my student Albert Fannjiang who just completed his Ph.D. at the Courant Institute and will be going to UCLA next year. We have written two papers. One, entitled "Convection enhanced diffusion", will be submitted to the SIAM Journal on Applied Mathematics in the next week or two. The second paper with a similar title is in preparation and will probably go to Communications in Math. Phys.

Here is a brief description of the contents of the first paper.

The temperature of T of a weakly conducting fluid in $\mathbb{R}^2$ satisfies the heat equation

$$\frac{\partial T}{\partial t} = \varepsilon \Delta T + \mathbf{u} \cdot \nabla T, \tag{1}$$

with $T(0,x,y) = T_0(x,y)$ given. Here $\mathbf{u}(x,y) = (u(x,y), v(x,y))$ is the fluid velocity which we assume incompressible

$$\nabla \cdot \mathbf{u} = 0$$

and $\varepsilon > 0$ is the molecular diffusivity which we assume small. We are interested in velocity fields that represent convective flow, as for example in Benard convection. Since $\mathbf{u}$ is incompressible there is a stream function $H(x,y)$ such that

$$\nabla^{\perp} H = (-H_y, H_x) = \mathbf{u} \tag{2}$$

A typical convective or cellular flow is the one given by

$$H(x,y) = \sin x \in y \tag{3}$$

The stream lines of the periodic flow are given by $H(x,y) =$ constant. We are interested in the *effective diffusivity* of the fluid and its behavior as the molecular diffusivity ε tends to zero.

The effective diffusivity is defined by

$$\sigma_\varepsilon^* = \lim_{t \uparrow \infty} \frac{1}{t} \iint (x^2 + y^2) T(t,x,y)\, dx\, dy \tag{4}$$

when the initial function T_0 is the delta function at the origin. With this initial function, $T(t,x,y)$ is the probability density of a test particle diffusing in the flow and (4) says that when t is large the means quare displacement of the particle behaves like $\sigma_\varepsilon^* t$.

We are interested in the behavior of σ_ε^* as $\varepsilon \to 0$. It has been known for some time, from a boundary layer analysis, that when H is given by (3) then

$$\sigma_\varepsilon^* \sim c^* \sqrt{\varepsilon} \tag{5}$$

as ε tends to zero and we can also characterize the constant c^*. The asymptotic relation (5) is the simplest example of convection ehanced diffusion because the effective diffusivity σ_ε^* is much larger than the molecular diffusivity ε. The enhancement is due to the convective flow with the stream function (3). Flows with stream functions

$$H(x,y) = \sin x \sin y + \delta \cos x \cos y, \tag{6}$$

with $0 \leq \delta \leq 1$ are also considered. They give rise to cellular flows with open channels in them.

Our aim in this paper is to present an analysis of the effective diffusivity of a passive scalar in a convective flow by variational methods, avoiding thus direct boundary layer analysis. This is important because boundary layer analysis becomes too complicated to be useful when the flow $\mathbf{u}$ is more complex than simple cellular flow or cellular flow with channels. This work is done jointly with Albert Fannjiang.

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Research Report
Michael J. Shelley

The year that I spent at IAS (1991-1992), as part of the special year in fluid turbulence, has been very productive for me. Work on various topics, mainly in fluid mechanics, has reached completion, progressed, or been initiated. In particular, three papers have been completed during my stay.

1. *The Collapse of Axisymmetric Vortex Sheets with Swirl*, with R. Caflisch and X. Li (UCLA), preprint. This work concerns both the theory and computation of instability and nonlinear evolution of axisymmetric vortex sheets.

2. *A Numerical Study of the Effect of Surface Tension and Noise on an Expanding Hele-Shaw Bubble*, with W. Dai (Chicago), submitted to Physics of Fluids A, 1992. This work concerns the interaction of surface tension in the selection of patterns in Hele-Shaw flows.

3. *Density driven pinching in the Hele-Shaw cell*, with R. Goldstein and A. Pesci (Princeton), to appear in the Proceedings of the NATO workshop on Singularities in Fluids and Optics, Crete, July 1992. This work concerns the modeling and computation of interface pinching, such as is observed in the formation of fluid droplets. A more comprehensive paper on this topic is currently in progress. This work has also branched into other areas, such as the partial inclusion of inertial effects in Hele-Shaw flows.

Work on other projects has also progressed or been initiated:

1. The study of density stratified boundary layers. This includes work on interfacial models of infinite Prandtl number boundary layers, with W. Dai (Chicago), and comptuations of stratified boundary layers in the regime of "hard" turbulence. the second work is being done with T. Hou (Courant) and J. Lowengrub (Minnesota), two other participants in the special year. The aim of these works is to understand the experimental observations of the Libchaber fluids group at Princeton.

2. The interaction of nematic fluids with electric fields. This is work done with D. McLaughlin and D. Muraki (Princeton applied math), and E. Brayn (Princeton physics). We are developing a theoretical description of the self-focusing of laser beams in nematic fluids. The governing equations have been derived; they are nonlinear Schrodinger equations, whose nonlinear index of refraction is coupled to the mechanical response of the director field in the fluid. We have also developed numerical methods for solving these equations, as well as studied their asymptotics.

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3. Numerical methods for computing the motion of fluid interfaces under surface tension. This is joint with T. Hou and J. Lowengrub. Surface tension is a fundamental regularizing mechanism in fluid interface motion, but is very difficult to account for in computation. Its inclusion leads to extremely stiff nonlinear systems. We have developed new methods to compute such flows which draw heavily on hard analysis of the equations of motion and a clever reformulation of the problem.